STUDENT'S NAME: ______________________________

TEACHER'S NAME: ______________________________

HURLSTONE AGRICULTURAL HIGH SCHOOL

2021

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 2

General Instructions

- Preparation time – 10 minutes
- Working time – 3 hours
- Scanning & uploading time – 1 hour
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided at the back of this paper
- In Questions in Section II, show all relevant mathematical reasoning and/or calculations

Total marks: 100

Section I – 10 marks (pages 2 – 4)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 5 – 10)

- Attempt Questions 11 – 16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks

Attempt Questions 1 – 10.

Allow about 15 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 10.

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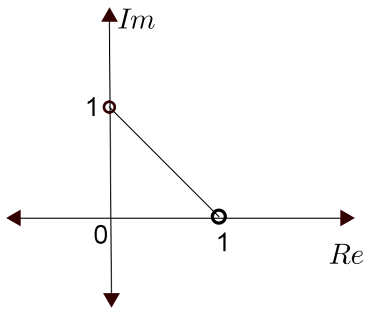
1. Let $z = a + ib$, where a and b are real, non-zero numbers.

   If $z + \frac{1}{z} \in \mathbb{R}$ which of the following must be true?

   A. $\arg z = \frac{\pi}{4}$ B. $a = -b$

   C. $a = b$ D. $|z| = 1$

2. The locus of z is displayed on the Argand diagram below.



   Which of the following is the equation of the locus of z?

   A. $\arg\left(\frac{z-i}{z-1}\right) = 0$ B. $\arg\left(\frac{z-i}{z-1}\right) = -\pi$

   C. $\arg\left(\frac{z+i}{z+1}\right) = 0$ D. $\arg\left(\frac{z-i}{z-1}\right) = \pi$

3. What is the smallest positive value of θ such that $e^{i\theta} \times e^{2i\theta} = i$?

   A. $\frac{\pi}{12}$ B. $\frac{\pi}{6}$

   C. $\frac{\pi}{3}$ D. $\frac{5\pi}{6}$

4. What is the approximate size of the angle between the vectors $\begin{bmatrix} \mathbf{2} \\ \mathbf{-3} \\ \mathbf{1} \end{bmatrix}$ **and** $\begin{bmatrix} \mathbf{1} \\ \mathbf{2} \\ \mathbf{-1} \end{bmatrix}$?

A. 57° B. 93°

C. 123° D. 158°

5. Given the vectors $\underset{\sim}{p} = 3\underset{\sim}{i} - 4\underset{\sim}{j} + 12\underset{\sim}{k}$ and $\underset{\sim}{q} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, which of the following shows the projection of $\underset{\sim}{p}$ on $\underset{\sim}{q}$?

A. $-\frac{14}{3}\left(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$ B. $-\frac{14}{9}\left(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}\right)$

B. $-\frac{14}{13}\left(3\underset{\sim}{i} - 4\underset{\sim}{j} + 12\underset{\sim}{k}\right)$ D. $-\frac{14}{169}\left(3\underset{\sim}{i} - 4\underset{\sim}{j} + 12\underset{\sim}{k}\right)$

6. Consider the statement "I will clean my room and do my homework"

If the above statement is **FALSE**, which of the following statements must be true?

A. I will not clean my room and not do my homework.

B. I will clean my room and not do my homework.

C. I will not clean my room and will do my homework.

D. I will not clean my room or I will not do my homework.

7. A student wants to prove that there is an infinite number of prime numbers. To prove this statement by contradiction, what assumption would the student start their proof with?

A. There is only one prime number that is even.

B. There is an infinite number of primes.

C. There is a finite number of primes.

D. All prime numbers are less than 100

8. Assume that a and b are positive real numbers with $a > b$.

Which of the following might be false?

A. $\dfrac{1}{a-b} > 0$

B. $\dfrac{a}{b} - \dfrac{b}{a} > 0$

C. $a + b > 2b$

D. $2a > 3b$

9. If $f(x)$ is a non-zero odd function with period π, which of the following statements is false?

A. $\int_{0}^{2\pi} f(x)\,dx = 0$

B. $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx = 2\int_{0}^{\frac{\pi}{2}} f(x)\,dx$

C. $\int_{0}^{\pi} f(x)\,dx = -\int_{0}^{\pi} f(-x)\,dx$

D. $\int_{a}^{a+\pi} f(x)\,dx = \int_{0}^{\pi} f(x)\,dx$ for any real number a.

10. Which of the following expressions is equal to $\displaystyle\int \frac{dx}{x\left(\log_e x\right)^2}$?

A. $\dfrac{1}{\log_e x} + c$

B. $\dfrac{1}{\left(\log_e x\right)^3} + c$

C. $\log_e\left(\dfrac{1}{x}\right) + c$

D. $-\dfrac{1}{\log_e x} + c$

End of Section 1

Section II

90 marks

Attempt Questions 11 – 16.

Allow about 2 hours and 45 minutes for this section.

Answer each question in a separate writing booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Use a separate writing booklet. **Marks**

(a) Let $z = 1 + i$ and $w = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$.

(i) Give the answer to $z + w$ in any suitable form. **2**

(ii) Express z in polar form. **1**

(iii) Hence or otherwise, express $(w\overline{z})^8$ also in polar form. **2**

(b) Solve $z^4 = (z-1)^4$ **4**

(c) (i) Use an Argand diagram to sketch the region for which:

$$|z| \leq 4\sqrt{2} \text{ and } \frac{\pi}{4} \leq \arg(z) \leq \frac{\pi}{3}$$

Include in your diagram the point in this region with the greatest real component.

Label this point P.

Also label the point in this region with the greatest imaginary component with a Q. **2**

(ii) Find the complex number represented by point P. **1**

(iii) Write, in the form $x + iy$, the complex number represented by the point Q. **1**

(iv) Determine whether the point representing the number $3 + 5i$ lies within this region. **2**

Question 12 (15 marks) Use a separate writing booklet. **Marks**

(a) P is the point $(4,-6,\ 3)$. Calculate the distance of P from:

(i) the origin **1**

(ii) the x-axis **1**

(b) 2 vectors are of the form: $\underset{\sim}{a}=\begin{bmatrix}\lambda\\1\\2\end{bmatrix}$ $\underset{\sim}{b}=\begin{bmatrix}\lambda-1\\2\\-4\end{bmatrix}$

Find all possible vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ that are perpendicular to each other. **2**

(c) The equations of two lines are:

$\underset{\sim}{r}_1=\begin{pmatrix}2\\-3\\1\end{pmatrix}+\lambda\begin{pmatrix}1\\4\\1\end{pmatrix}$ and $\underset{\sim}{r}_2=\begin{pmatrix}-4\\6\\2\end{pmatrix}+\mu\begin{pmatrix}4\\-11\\3\end{pmatrix}$

(i) Explain why these lines are not parallel. **1**

(ii) Determine whether these lines intersect or not. **3**

(d) The co-ordinates of 3 points are $A=(1,0,5)$ $B=(-1,2,4)$ $C=(3,5,2)$.

(i) Express the vector $\overrightarrow{AB}$ in the form $x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k}$ **1**

(ii) Find the co-ordinates of the point D such that $ABCD$ is a parallelogram. **2**

(iii) Prove that $ABCD$ is a rectangle. **2**

(e) Describe the behaviour of the following curve as t increases, given that $t\geq 0$

$\underset{\sim}{r}(t)=\cos(t)\underset{\sim}{i}+\sin(t)\underset{\sim}{j}+(t)\underset{\sim}{k}$ **2**

Question 13 (15 marks) Use a separate writing booklet. **Marks**

(a) Consider the statement: $\forall\, x \in \mathbb{R}, \exists\, y \in \mathbb{R}$ such that $x^2 + y^2 = 2$.

Is the statement true or false?

Justify your answer. **1**

(b) Given that *m* and *n* are integers, consider the following statement:

"If *mn* is odd, then *m* and *n* are odd".

(i) Write down the contrapositive of this statement. **1**

(ii) Prove the initial statement is true by proving its contrapositive. **2**

(c) Prove that $\log_2 5$ is an irrational number. **3**

(d) (i) By considering the cases where a positive integer *k* is even $(\boldsymbol{k = 2x})$ and odd $(\boldsymbol{k = 2x+1})$, show that $\boldsymbol{k^2 + k}$ is always even. **2**

(ii) Using the result in part (i), prove, by mathematical induction, that for all positive integral values of n, $\boldsymbol{n^3 + 5n}$ is divisible by 6. **3**

(e) Given $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$,

Show that $\dfrac{x+y+z}{3} \geq \sqrt[3]{xyz}$ for x > 0, y > 0 and z > 0. **3**

Question 14 (15 marks) Use a separate writing booklet. **Marks**

(a) (i) Find real numbers a, b and c, such that **2**

$$\frac{5}{x^2(2-x)} \equiv \frac{ax+b}{x^2} + \frac{c}{2-x}.$$

(ii) Hence, or otherwise, find $\displaystyle\int \frac{20}{x^2(2-x)}\,dx$ **2**

(b) Find $\displaystyle\int \frac{dx}{\sqrt{8-2x-x^2}}\,dx$ **3**

(c) By using a suitable substitution, evaluate $\displaystyle\int_{3\sqrt{2}}^{6} \frac{1}{x^2\sqrt{x^2-9}}\,dx$ **3**

(d) Find $\displaystyle\int \cos^5 x\,dx$ **3**

(e) Find $\displaystyle\int \frac{dx}{1+\cos x}$ **2**

Question 15 (15 marks) Use a separate writing booklet. **Marks**

(a) The straight line L has the vector equation $\underset{\sim}{r} = 2\underset{\sim}{i} + \underset{\sim}{j} - \underset{\sim}{k} + \lambda\left(-\underset{\sim}{i} + 3\underset{\sim}{j} + \underset{\sim}{k}\right)$.

The point O is at the origin.

(i) The point $P(0, y, z)$ lies on L. Find the values of y and z. **2**

(ii) The point Q lies on L such that OQ is perpendicular to L.

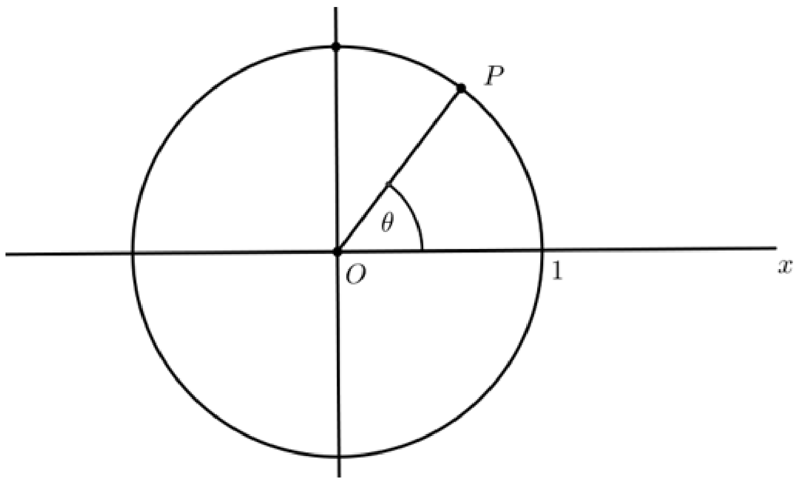
Find the coordinates of Q. **2**

(iii) Find the area of ΔOPQ. **2**

(b) In the Argand diagram below, point P lies in the first quadrant on the unit circle.

P represents the complex number ω and $z = \omega$ is a root of $z^5 - 1 = 0$.

$\angle POx = \theta$.



(i) Show that $\theta = 72^\circ$. **1**

(ii) Show that $1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$. **2**

(iii) If $a = \omega + \omega^4$ and $b = \omega^2 + \omega^3$ show that $a + b = -1$ and $ab = -1$. **2**

(iv) Show that $(a - b)^2 = 5$. **1**

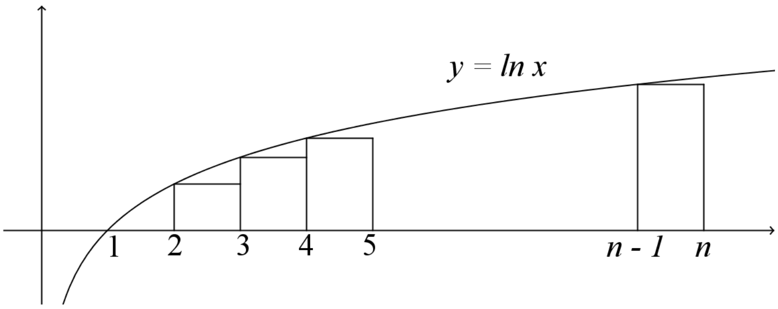
(v) Given $(a - b) > 0$, find the exact value of $\cos 72^\circ$. **3**

Question 16 (15 marks) Use a separate writing booklet. **Marks**

(a) (i) Use integration by parts to evaluate $\int_1^n \ln x \, dx$. **2**

(ii) Consider the curve $y = \ln x$.

Rectangles of width 1 unit are drawn below the curve to approximate the area under the curve for $1 \le x \le n$ as shown below.



Show that the sum of the areas of the rectangles is $\ln\left((n-1)!\right)$ units^2. **2**

(iii) Hence, or otherwise prove that $\ln\left((n-1)!\right) < n\ln n - n + 1 < \ln\left(n!\right)$. **2**

(iv) Prove that $(n-1)! < n^n e^{1-n} < n!$ **1**

(b) Let $I_n = \int_0^1 x^n \sqrt{1-x} \, dx, \quad n = 0, 1, 2, \ldots$

(i) Prove that $I_n = \dfrac{2n}{2n+3} I_{n-1}, \; n \ge 1$ **3**

(ii) Evaluate $\int_0^1 x^3 \sqrt{1-x} \, dx$. **2**

(iii) Prove that $I_n = \dfrac{n!\,(n+1)!\,4^{n+1}}{(2n+3)!}$ **3**

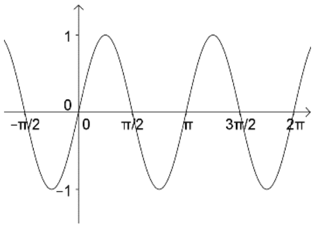
End of Examination.

Year 12 Higher School Certificate	Mathematics Extension 2	Task 4 2021
Questions 1 - 10	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings
MEX12-3 uses vectors to model and solve problems in two and three dimensions
MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems
MEX12-5 applies techniques of integration to structured and unstructured problems

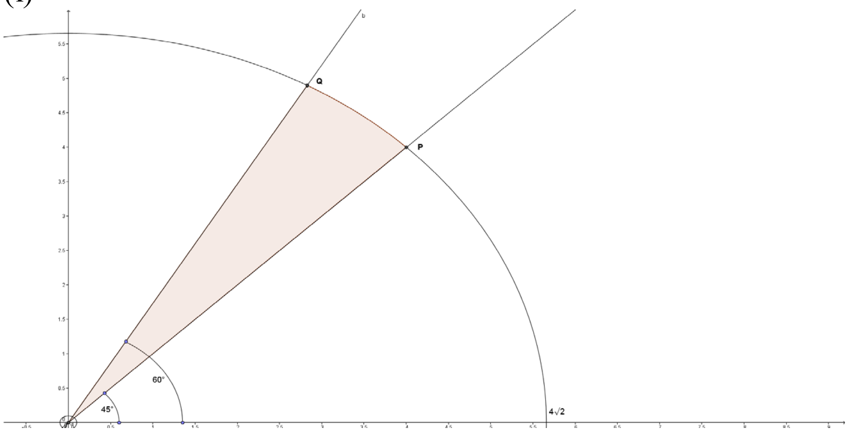
Outcome	Solutions	1 mark each
MEX12-4	**1. D** $z = rcis\theta \qquad \frac{1}{z} = z^{-1} = r^{-1}cis(-\theta)$ Since their arguments are opposites, adding the numbers' vectors together will create symmetry above and below the horizontal. The only way this can result in an imaginary part equal to zero is if the two moduli are equal. $\text{mod}(z) = \text{mod}\left(\frac{1}{z}\right) \quad \rightarrow \text{mod}(z) = 1$	
MEX12-4	**2. B** The line is between the end-points, so eliminate A, C. Now decide whether arg(numerator) minus arg(denominator) is positive or negative. Arg(z-i) measures the direction from (0,1). Arg(z-1) measures the direction from (1,0).	
MEX12-4	**3. B** If $e^{i\theta} \times e^{2i\theta} = i$, then $e^{3i\theta} = i$ $\therefore \cos 3\theta + i\sin 3\theta = i$ Matching real & imaginary parts, $\therefore \cos 3\theta = 0, \ \sin 3\theta = 1$ Solving $3\theta = \frac{\pi}{2}$ for smallest positive value of θ, $\theta = \frac{\pi}{6}$	
MEX12-3	**4. C** $\cos\theta = \frac{\underset{\sim}{a} \bullet \underset{\sim}{b}}{\lvert\underset{\sim}{a}\rvert\lvert\underset{\sim}{b}\rvert} = \frac{-5}{84} \qquad \rightarrow \theta = 123°$	
MEX12-3	**5. B** $\underset{\sim}{p}$ "on" $\underset{\sim}{q}$ rules out A. $\frac{\underset{\sim}{p} \bullet \underset{\sim}{q}}{\lvert\underset{\sim}{q}\rvert^2} = \frac{-14}{9}$	
MEX12-2	**6. D**	
MEX12-2	**7. C** Contradicting an infinite number of primes is that there is a finite number of primes	

MEX12-2	**8. D** a and b are positive and $a > b$ If $a=3,\ b=2$ False! So D. OR Not C as adding b to both sides of $a>b$ gives $a+b>2b$ so always true $\frac{a}{b}-\frac{b}{a}=\frac{a^2-b^2}{ab}=\frac{(a-b)(a+b)}{ab}$ which is positive so not B $\frac{1}{a-b}=1\div\text{positive}=\text{positive}$, so not A
MEX12-5	**9. B** As odd, must have point symmetry about the origin and as period π, consider $y=\sin 2x$.  $\int_0^{2\pi} f(x)\,dx=0$ Equal areas above & below x axis A is true $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} f(x)\,dx=2\int_0^{\frac{\pi}{2}} f(x)\,dx$ LHS = 0, RHS $\neq 0$ B is false $-\int_0^{\pi} f(-x)\,dx=-\int_0^{\pi} -f(x)\,dx=\int_0^{\pi} f(x)\,dx$ as odd function. C true $\int_a^{a+\pi} f(x)\,dx=\int_0^{\pi} f(x)\,dx$ Equal integrals for any section of length π units as areas above & below x axis. D true
MEX12-5	**10. D** Let $u=\log_e x$ $\therefore \frac{du}{dx}=\frac{1}{x}$ & $\therefore du=\frac{dx}{x}$ $\therefore \int \frac{dx}{x(\log_e x)^2}=\int \frac{du}{u^2}=\int u^{-2}du$ $=\frac{u^{-1}}{-1}+c \quad =\frac{-1}{\log_e x}+c$

Outcomes Addressed in this Question

MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Outcome	Solutions	Marking Guidelines
(a) **MEX12-4**	(i) $z+w=1+i+\left(\frac{\sqrt{3}}{2}+\frac{i}{2}\right)$ $=\left(1+\frac{\sqrt{3}}{2}\right)+\frac{3i}{2}$ or equivalent.	**(a) (i) 2 marks**: Both modulus and principal argument correct. **1 mark**: Correct conversion of one number's format.
	(ii) $z=\sqrt{2}\,e^{\frac{i\pi}{4}}=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$	**(ii) 1 mark**: Correct answer.
	(iii) $(w\bar{z})^8=\left(e^{\frac{i\pi}{6}}.\sqrt{2}\,e^{\frac{-i\pi}{4}}\right)^8$ $=\left(\sqrt{2}\right)^8\left(e^{\frac{-i\pi}{12}}\right)^8$ $=16e^{\frac{-2i\pi}{3}}$ $=16\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)$	**(iii) 2 marks:** Correct solution (CFPA (ii)). **1 mark:** Significant relevant progress shown.
(b) **MEX12-4**	$z^4=(z-1)^4 \quad\rightarrow\quad \left(\frac{z}{z-1}\right)^4=1$ $\therefore \frac{z}{z-1}=\pm1,\pm i$ $\frac{z}{z-1}=1 \quad\rightarrow$ No solution $\frac{z}{z-1}=-1 \quad\rightarrow z=\frac{1}{2}$ $\frac{z}{z-1}=i \quad\rightarrow z=\frac{-1}{1-i}\times\frac{1+i}{1+i}=\frac{1-i}{2}$ $\frac{z}{z-1}=-i \quad\rightarrow z=\frac{i}{1+i}\times\frac{1-i}{1-i}=\frac{1+i}{2}$ Alternatively, you could use polynomial methods to get the factorisation $(2z-1)\left(2z^2-2z+1\right)=0$ And solve using the quadratic formula, sum of roots or equivalent. Another alternative: Equate to zero, and factorise as a difference of 2 squares. Very elegant method with quick solution.	**(b)** **4 marks:** Complete solution. **3 marks**: One error in solving method. **2 marks:** Significant progress towards correct solution. **1 mark**: Some relevant progress.

(c) **MEX12-4**	(i)  (ii) $P = 4\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$ or equivalent (iii) $Q = 4\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ $= 2\sqrt{2} + 2i\sqrt{6}$ (iv) $3^2 + 5^2 = 34 > 32$ therefore the distance from the origin is beyond the radius of the sector. It will not lie in the region.	**(c)** **(i) 2 marks**: Diagram includes correct boundaries, and correct sector radius with shading. **1 mark**: Significant relevant progress. **(ii) 1 mark:** Correct answer. **(iii) 1 mark:** Correct answer in required form. **(iv) 2 marks:** Correct answer with correct justification. **1 mark:** One component correct from above.

Outcomes Addressed in this Question

MEX12-3 uses vectors to model and solve problems in two and three dimensions

Outcome	Solutions	Marking Guidelines
(a) **MEX12-3**	Distance from: (i) origin $=\sqrt{4^2+(-6)^2+3^2}=\sqrt{61}$ units (ii) x-axis $=\sqrt{(-6)^2+3^2}=\sqrt{45}$ units	**(a) (i) 1 mark**: Correct answer. **(ii) 1 mark**: Correct answer.
(b) **MEX12-3**	Dot product = 0 $\lambda(\lambda-1)+1(2)+2(-4)=0$ $\lambda^2-\lambda-6=0$ $\lambda=-2,3$ When $\lambda=-2$ $\quad \underset{\sim}{a}=\begin{pmatrix}-2\\1\\2\end{pmatrix}$ $\quad \underset{\sim}{b}=\begin{pmatrix}-3\\2\\-4\end{pmatrix}$ When $\lambda=3$ $\quad \underset{\sim}{a}=\begin{pmatrix}3\\1\\2\end{pmatrix}$ $\quad \underset{\sim}{b}=\begin{pmatrix}2\\2\\-4\end{pmatrix}$	**(b)** **2 marks:** Both correct pairs. **1 mark**: One correct pair from correct λ, or 2 correct pairs from incorrect λ
(c) **MEX12-3**	(i) Direction vectors are not scalar multiples of each other. $\begin{pmatrix}1\\4\\1\end{pmatrix}\neq k\begin{pmatrix}4\\-11\\3\end{pmatrix}$ (ii) $\underset{\sim}{r_1}=\underset{\sim}{r_2}$ $\quad 2+\lambda=-4+4\mu.........\langle 1\rangle$ $-3+4\lambda=6-11\mu......\langle 2\rangle$ $1+\lambda=2+3\mu.............\langle 3\rangle$ Solving $\langle 1\rangle$ and $\langle 3\rangle$ simultaneously gives $\lambda=22;\ \mu=7$ Substitution into $\langle 2\rangle$ gives $LHS = 85$; $RHS = 71$. Not satisfied. Therefore the lines do **not** intersect. **Note:** pairing the 3 equations differently will give different values for λ and μ	**(c) (i) 1 mark:** Correct statement explaining which are the direction vectors for this eg. **(ii) 3 marks:** Correct solution with justification. **2 marks:** Significant progress towards correct solution. **1 mark:** Relevant progress.

(d) **MEX 12-3**	(i) $\overrightarrow{AB}=\begin{pmatrix}-2\\2\\-1\end{pmatrix}=-2\underset{\sim}{i}+2\underset{\sim}{j}-\underset{\sim}{k}$ (ii) $\overrightarrow{DC}=\overrightarrow{AB}$ $\therefore D=\begin{pmatrix}k\\l\\m\end{pmatrix}=\begin{pmatrix}3\\5\\2\end{pmatrix}-\begin{pmatrix}-2\\2\\-1\end{pmatrix}=\begin{pmatrix}5\\3\\3\end{pmatrix}$ (iii) $\overrightarrow{AB}\bullet\overrightarrow{AD}=\begin{pmatrix}-2\\2\\-1\end{pmatrix}\bullet\begin{pmatrix}4\\3\\-2\end{pmatrix}=-8+6+2=0$ $\therefore AB\perp AD$ *ABCD* is a rectangle because two adjacent sides are perpendicular. **Note:** Alternatively, you could prove that diagonals are equal in length.	**(d)(i) 1 mark:** Correct answer in format required. **(ii)** **2 marks:** Correct solution. **1 mark:** Substantial progress. **(iii) 2 marks:** Correct solution and geometrical property described. **1 mark:** Substantial progress.
(e) **MEX 12-3**	The curve "begins" at (1, 0, 0) when t=0 and rises in the form of a <u>circular spiral with radius 1</u>, above the circle $x^2+y^2=1$ and about the *z*-axis. It rises by 2π units every revolution.	**(e) 2 marks:** Description including the underlined point and another feature. **1 mark:** Partial description.

Year 12	Mathematics Extension 2	Ass Task 4 2021 HSC
Question No. 13	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings

Part / Outcome	Solutions	Marking Guidelines
(a)	FALSE (counter-example below) $x^2+y^2=2$ if $x=2$ $4+y^2=2$ $y^2=-2$ $\quad \therefore$ No real solution	<u>1 mark</u>: correct solution *(Provides correct counter example)*
(b)(i)	"If *mn* is odd, then *m* and *n* are odd" Contrapositive: "If *m* or *n* are even, then *mn* is even"	<u>1 mark</u>: correct solution
(b)(ii)	If m is even and n is even, $m=2p,\ n=2q$ $\quad p,q$ are integers $mn=2p\times 2q$ $=2(2pq)$ $\quad \therefore mn$ is even If m is even and n is odd, $m=2p,\ n=2q+1$ $\quad p,q$ are integers $mn=2p\times(2q+1)$ $=2(p(2q+1))$ $=2k$ $\quad \therefore mn$ is even	<u>2 marks</u> : correct solution <u>1 mark</u> : substantially correct solution NB: 3cases to consider… m even, n odd; m odd, n even; both even. This boils down to two cases to consider: Either one even or both even
(c)	Suppose (by way of contradiction) that $\log_2 5$ is rational $\log_2 5=\frac{p}{q}$ where $\begin{cases} p,q \text{ are integers with no common factors} \\ \text{and } q\geq 1\end{cases}$ ie $2^{\frac{p}{q}}=5$ $2^p=5^q$ $\quad$ (raising both side to the q^{th} power) Now, LHS is even and RHS is odd This is a contradiction $\therefore \log_2 5$ is *not* rational ie $\log_2 5$ is an irrational number	<u>3 marks</u>: correct proof <u>2 marks</u>: substantially correct solution (*eg determines* $2^p=5^q$) <u>1 mark</u>: *attempts* proof by contradiction

	Question 13 continued…	
(d)(i)	If k is even, ie $k = 2x$ then $k^2 + k = (2x)^2 + 2x$ $= 4x^2 + 2x$ $= 2(2x^2 + x)$ $= 2m \quad \therefore$ even If k is odd, ie $k = 2x + 1$ then $k^2 + k = (2x+1)^2 + 2x + 1$ $= 4x^2 + 4x + 1 + 2x + 1$ $= 4x^2 + 6x + 2$ $= 2(2x^2 + 3x + 1)$ $= 2n \quad \therefore$ even	<u>2 marks</u>: correct solution *(showing both results)* <u>1 mark</u>: substantially correct solution *(showing one result, or equivalent merit)*
(d)(ii)	"$n^3 + 5n$ is divisible by 6" Show true for $n = 1$ $1^3 + 5(1) = 6,$ which is divisible by 6 Assume true for $n = k$ ie $k^3 + 5k = 6M,$ for integral M Prove true for $n = k + 1$ ie $(k+1)^3 + 5(k+1) = 6N,$ for integral N now, LHS $= (k+1)^3 + 5(k+1)$ $= k^3 + 3k^2 + 3k + 1 + 5k + 5$ $= k^3 + 5k + 3k^2 + 3k + 6$ $= 6M + 3(k^2 + k) + 6$ from assumption $= 6M + 3(2p)^* + 6$ *from (i) $= 6(M + p + 1)$ $= 6N$ ie divisible by 6 $\therefore$ true by mathematical induction	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution <u>1 mark</u>: partially correct solution

<table>
<tr><td></td><td>Question 13 continued…</td><td></td></tr>
<tr><td>(e)</td><td>

Using $(x-y)^2 \geq 0 \Rightarrow x^2+y^2-2xy \geq 0$

$$x^2+y^2 \geq 2xy \quad \ldots(1)$$
$$x^2+z^2 \geq 2xz \quad \ldots(2)$$
$$y^2+z^2 \geq 2yz \quad \ldots(3)$$

Adding (1), (2), (3) gives

$$2(x^2+y^2+z^2) \geq 2(xy+yz+xz)$$
$$x^2+y^2+z^2 \geq xy+yz+xz$$

and given that $x, y, z > 0,$

then $xy+yz+xz > 0$

and so $x^2+y^2+z^2-xy-yz-xz \geq 0$

also $x+y+z > 0$

Now,

$$x^3+y^3+z^3-3xyz = (x+y+z)(x^2+y^2+z^2-xy-yz-xz) \geq 0$$
$$x^3+y^3+z^3-3xyz \geq 0$$

let $a = x^3$, $b = y^3$, $c = z^3$

$$a+b+c-3\sqrt[3]{a}\times\sqrt[3]{b}\times\sqrt[3]{c} \geq 0$$
$$a+b+c \geq 3\sqrt[3]{abc}$$
$$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$$
$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}$$

NOTE: a common line of working in candidates solutions was the following:

$$x^2+y^2+z^2-xy-yz-xz \geq 0$$
$$(x+y+z)(x^2+y^2+z^2-xy-yz-xz) \geq 0$$

this can only be true if $x+y+z \geq 0$, and this condition, and the reason it is true needs to be stated for the rest of the proof to hold as true.

Alternate solution:

Using $\left(\sqrt{x}-\sqrt{y}\right)^2 \geq 0 \Rightarrow x+y-2\sqrt{xy} \geq 0$

$$x+y \geq 2\sqrt{xy} \quad \ldots(1)$$
$$w+z \geq 2\sqrt{wz} \quad \ldots(2)$$

</td><td>

<u>3 marks:</u> Correct solution.

<u>2 marks:</u> Makes significant progress towards the solution.

1 Mark: Makes some progress towards the solution such as using $x^2+y^2 \geq 2xy$

</td></tr>
</table>

	adding (1) and (2)… $x+y+w+z \geq 2\left(\sqrt{xy}+\sqrt{wz}\right)$ $\geq 2\left(2\sqrt{\sqrt{xy}\times\sqrt{wz}}\right)$ so $x+y+w+z \geq 4\times\sqrt[4]{xywz}$ let $x+y+z=3w \quad \Rightarrow \quad w=\dfrac{x+y+z}{3}$ so now, $4w \geq 4\times\sqrt[4]{wxyz}$ $w \geq \sqrt[4]{wxyz}$ $w^4 \geq wxyz$ $w^3 \geq xyz$ $w \geq \sqrt[3]{xyz}$ $\dfrac{x+y+z}{3} \geq \sqrt[3]{xyz}$	

<table>
<tr><td>Year 12 Higher School Certificate   Mathematics Extension 2   Task 4 2021
Question No. 14   Solutions and Marking Guidelines</td></tr>
<tr><td>Outcomes Addressed in this Question</td></tr>
<tr><td>MEX12-5 applies techniques of integration to structured and unstructured problems</td></tr>
</table>

Solutions	Marking Guidelines
(a) (i) $\dfrac{5}{x^2(2-x)} \equiv \dfrac{ax+b}{x^2}+\dfrac{c}{2-x}$ $5 \equiv (ax+b)(2-x)+cx^2$ $5 \equiv (c-a)x^2+(2a-b)x+2b$ $\therefore 2b=5,\ 2a-b=0,\ c-a=0$ by matching like coefficients $\therefore b=\dfrac{5}{2},\ a=\dfrac{5}{4},\ c=\dfrac{5}{4}.$	2 marks : correct solution 1 mark : significant progress towards correct solution
(ii) $\displaystyle\int \frac{20}{x^2(2-x)}\,dx = \int 4\times\frac{5}{x^2(2-x)}\,dx$ $\displaystyle = 4\int \frac{\frac{5}{4}x+\frac{5}{2}}{x^2}+\frac{\frac{5}{4}}{2-x}\,dx$ $\displaystyle = \int \frac{5x+10}{x^2}+\frac{5}{2-x}\,dx$ $\displaystyle = \int \frac{5}{x}+10x^{-2}-5.\frac{-1}{2-x}\,dx$ $\displaystyle = 5\log\lvert x\rvert+\frac{10x^{-1}}{-1}-5\log\lvert 2-x\rvert+c$ $\displaystyle = 5\log\left\lvert\frac{x}{2-x}\right\rvert-\frac{10}{x}+c$ (b)	2 marks : correct solution 1 mark : significant progress towards correct solution
$\displaystyle\int \frac{dx}{\sqrt{8-2x-x^2}}\,dx = \int \frac{dx}{\sqrt{-(x^2+2x-8)}}\,dx$ $\displaystyle = \int \frac{dx}{\sqrt{-(x^2+2x+1-9)}}\,dx$ $\displaystyle = \int \frac{dx}{\sqrt{-((x+1)^2-9)}}\,dx$ $\displaystyle = \int \frac{dx}{\sqrt{9-(x+1)^2}}\,dx$ $\displaystyle = \sin^{-1}\left(\frac{x+1}{3}\right)+c$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution
(c) For $\displaystyle\int_{3\sqrt{2}}^{6} \frac{1}{x^2\sqrt{x^2-9}}\,dx$, let $x=3\sec\theta$ for $0<\theta<\dfrac{\pi}{2}.$ $\displaystyle \therefore \frac{dx}{d\theta}=3\sec\theta\tan\theta$	3 marks : correct solution 2 marks: substantial progress towards correct solution

When $x=6$, $\sec\theta=2$, $\theta=\dfrac{\pi}{3}$, when $x=3\sqrt{2}$, $\sec\theta=\sqrt{2}$, $\theta=\dfrac{\pi}{4}$. $$\therefore \int_{3\sqrt{2}}^{6}\frac{1}{x^2\sqrt{x^2-9}}dx=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{1}{9\sec^2\theta\sqrt{9\sec^2\theta-9}}\times 3\sec\theta\tan\theta\ d\theta$$ $$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\tan\theta\ d\theta}{9\sec\theta\sqrt{\sec^2\theta-1}}$$ $$=\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{\tan\theta\ d\theta}{9\sec\theta\sqrt{\tan^2\theta}}$$ $$=\frac{1}{9}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\frac{d\theta}{\sec\theta}$$ $$=\frac{1}{9}\int_{\frac{\pi}{4}}^{\frac{\pi}{3}}\cos\theta d\theta$$ $$=\frac{1}{9}\left[\sin\theta\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$ $$=\frac{\sqrt{3}-\sqrt{2}}{18}$$	1 mark : significant progress towards correct solution
(d) $\int\cos^5 x\ dx\ =\int\cos^4 x\cos x\,dx$ $$=\int\left(1-\sin^2 x\right)^2\cos x\,dx$$ $$=\int\left(1-2\sin^2 x+\sin^4 x\right)\cos x\,dx$$ $$=\int\left(\cos x-2\sin^2 x\cos x+\sin^4 x\cos x\right)dx$$ $$=\int\left(\cos x-2\sin^2 x\cos x+\sin^4 x\cos x\right)dx$$ $$=\sin x-\frac{2\sin^3 x}{3}+\frac{\sin^5 x}{5}+c$$	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution
(e) For $\displaystyle\int\frac{dx}{1+\cos x}$, let $t=\tan\dfrac{x}{2}$ $$\therefore \frac{dt}{dx}=\frac{1}{2}\sec^2\frac{x}{2}$$	2 marks: correct solution 1 mark: substantial progress towards solution

$$\therefore 2\frac{dt}{dx} = \tan^2\frac{x}{2} + 1$$ $$\therefore \frac{dx}{dt} = \frac{2}{t^2+1}$$ $$\therefore \int \frac{dx}{1+\cos x} = \int \frac{\frac{2\,dt}{t^2+1}}{1+\frac{1-t^2}{1+t^2}}$$ $$= \int \frac{2\,dt}{1+t^2+1-t^2}$$ $$= \int dt$$ $$= t + c$$ $$= \tan\frac{x}{2} + c$$	

Year 12	Mathematics Extension 2	Ass Task 4 2021 HSC
Question No. 15	Solutions and Marking Guidelines	

Outcomes Addressed in this Question

MEX12-3 uses vectors to model and solve problems in two and three dimensions
MEX12-4 uses the relationship between algebraic and geometric representations of complex numbers and complex number techniques to prove results, model and solve problems

Part / Outcome	Solutions	Marking Guidelines
(a)(i) **MEX12-3**	$P(0,y,z)$ satisfies $\underset{\sim}{r} = 2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}+\lambda(-\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k})$ x-coordinate... $2+(-\lambda)=0$ $\lambda = 2$ y-coordinate... $y = 1+\lambda(3)$ $=1+2(3)=7$ z-coordinate... $z=-1+\lambda(1)$ $=-1+2(1)=1$ $\therefore y=7,\ z=1$ ie $P(0,7,1)$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution (finds λ or shows some understanding)
(a)(ii) **MEX12-3**	$\overrightarrow{OQ}$ is perpendicular to $-\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k}$ so $(-\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k})\bullet(x\underset{\sim}{i}+y\underset{\sim}{j}+z\underset{\sim}{k})=0$ $-1\times x+3\times y+1\times z=0$ $-x+3y+z=0$ $Q(x,y,z)$ lies on $2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}+\lambda(-\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k})$ Equating the x, y and z coordinates: $-(2-\lambda)+3(1+3\lambda)+(-1+\lambda)=0$ $-2+\lambda+3+9\lambda-1+\lambda=0$ $\lambda=0$ $\overrightarrow{OQ}=2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}+0(-\underset{\sim}{i}+3\underset{\sim}{j}+\underset{\sim}{k})$ $=2\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$ $\therefore Q$ is $(2,1,-1)$	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution (finds $-x+3y+z=0$ or shows some relevant understanding)
(a)(iii) **MEX12-3**	$\angle OQP = 90° \Rightarrow$ need $\lvert\overrightarrow{OQ}\rvert$ and $\lvert\overrightarrow{QP}\rvert$ $\lvert\overrightarrow{OQ}\rvert=\sqrt{2^2+1^2+(-1)^2}=\sqrt{6}$ $\overrightarrow{QP}=\overrightarrow{OP}-\overrightarrow{OQ}$ $=(0-2)\underset{\sim}{i}+(7-1)\underset{\sim}{j}+(1--1)\underset{\sim}{k}$ $=-2\underset{\sim}{i}+6\underset{\sim}{j}+2\underset{\sim}{k}$ $\lvert\overrightarrow{QP}\rvert=\sqrt{(-2)^2+6^2+2^2}=\sqrt{44}$ $A=\frac{1}{2}\times\sqrt{6}\times\sqrt{44}$ $=\sqrt{66}$ or 8.12 square units	<u>2 marks</u>: correct solution <u>1 mark</u>: substantially correct solution

	Question 15 continued…	
(b)(i) **MEX12-4**	$\left(1\text{cis}72^\circ\right)^5 = \text{cis}360^\circ = 1$	1 mark: correctly show the result
(b)(ii) **MEX12-4**	ω is a root of $\quad z^5 - 1 = 0$ ie $\quad \omega^5 - 1 = 0$ $(\omega - 1)(1 + \omega + \omega^2 + \omega^3 + \omega^4) = 0$ so $\quad 1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$ *or…* sum of roots, $\quad \begin{aligned} 1 + \omega + \omega^2 + \omega^3 + \omega^4 &= -\frac{b}{a} \\ &= -\frac{0}{1} \\ &= 0 \end{aligned}$ *or…* sum of GP, $\quad \begin{aligned} 1 + \omega + \omega^2 + \omega^3 + \omega^4 &= \frac{1(\omega^5 - 1)}{\omega - 1} \\ &= \frac{(1-1)}{\omega - 1} \qquad (\omega^5 = 1) \\ &= 0 \end{aligned}$	2 marks: correct solution 1 mark: substantially correct solution
(b)(iii) **MEX12-4**	$\begin{aligned} a + b &= (\omega + \omega^4) + (\omega^2 + \omega^3) \\ &= 1 + \omega + \omega^2 + \omega^3 + \omega^4 - 1 \\ &= 0 - 1 \\ &= -1 \end{aligned}$ $\begin{aligned} ab &= (\omega + \omega^4)(\omega^2 + \omega^3) \\ &= \omega^3 + \omega^4 + \omega^6 + \omega^7 \\ &= \omega^3 + \omega^4 + \omega^5 \times \omega + \omega^5 \times \omega^2 \qquad (\omega^5 = 1) \\ &= \omega^3 + \omega^4 + \omega + \omega^2 \\ &= -1 \end{aligned}$	2 marks: correct solution 1 mark: substantially correct solution
(b)(iv) **MEX12-4**	$\begin{aligned} (a - b)^2 &= a^2 - 2ab + b^2 \\ &= a^2 + 2ab + b^2 - 4ab \\ &= (a + b)^2 - 4ab \\ &= (-1)^2 - 4(-1) \\ &= 5 \end{aligned}$	1 mark: correctly show the result

(b)(v) **MEX12-4**	*Question 15 continued…* $a+b=-1 \quad \ldots(1)$ $a-b=\sqrt{5} \quad \ldots(2)$ $2a=\sqrt{5}-1 \quad (1)+(2)$ $a=\dfrac{\sqrt{5}-1}{2}$ ie $\quad \omega+\omega^4=\dfrac{\sqrt{5}-1}{2}$ $\left(\cos 72^\circ+i\sin 72^\circ\right)+\left(\cos 72^\circ+i\sin 72^\circ\right)^4=\dfrac{\sqrt{5}-1}{2}$ $\left(\cos 72^\circ+i\sin 72^\circ\right)+\left(\cos\left(4\times 72^\circ\right)+i\sin\left(4\times 72^\circ\right)\right)=\dfrac{\sqrt{5}-1}{2}$ $\left(\cos 72^\circ+i\sin 72^\circ\right)+\left(\cos 288^\circ+i\sin 288^\circ\right)=\dfrac{\sqrt{5}-1}{2}$ $\left(\cos 72^\circ+i\sin 72^\circ\right)+\left(\cos 72^\circ-i\sin 72^\circ\right)=\dfrac{\sqrt{5}-1}{2}$ $2\cos 72^\circ=\dfrac{\sqrt{5}-1}{2}$ $\cos 72^\circ=\dfrac{\sqrt{5}-1}{4}$	<u>3 marks</u>: correct solution <u>2 marks</u>: substantially correct solution *(progress involving De Moivre's theorem)* <u>1 mark</u>: partially correct solution *(establishes $2a=\sqrt{5}-1$)*

Year 12 Higher School Certificate Mathematics Extension 2 Task 4 2021
Question No. 16 Solutions and Marking Guidelines

Outcomes Addressed in this Question

MEX12-2 chooses appropriate strategies to construct arguments and proofs in both practical and abstract settings
MEX12-5 applies techniques of integration to structured and unstructured problems

Outcome	Solutions	Marking Guidelines
MEX12-5	(a) (i)	2 marks : correct solution 1 mark : significant progress towards correct solution

$$\int_1^n \ln x\,dx = \int_1^n \ln x.1\,dx$$

$$= [\ln x.x]_1^n - \int_1^n x.\frac{1}{x}dx \qquad \text{using } \int u dv = uv - \int v du$$

$$= n\ln n - 0 - \int_1^n dx$$

$$= n\ln n - [x]_1^n$$

$$= n\ln n - (n-1) \quad = n\ln n - n + 1$$

Outcome	Solutions	Marking Guidelines
MEX12-5	(ii)	2 marks : correct solution 1 mark : significant progress towards correct solution
MEX12-2		

(ii) Let $f(x) = \ln x$.

As the width of each column is 1 unit,
Sum of the areas of the lower rectangles

$$= f(2)\times 1 + f(3)\times 1 + f(4)\times 1 + \ldots + f(n-1)\times 1$$

$$= \ln 2 + \ln 3 + \ln 4 + \ldots + \ln(n-1)$$

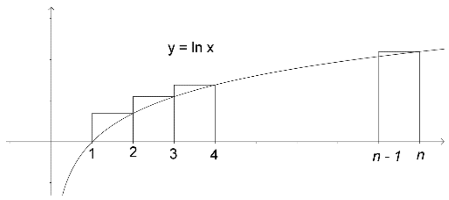
$$= \ln 1 + \ln 2 + \ln 3 + \ldots + \ln(n-1)$$

$$= \ln\{1.2.3\ldots(n-1)\}$$

$$= \ln((n-1)!)\ \text{u}^2$$

Outcome	Solutions	Marking Guidelines
MEX12-2	(iii)	2 marks: correct solution demonstrating clarity 1 marks: substantial progress towards solution

(iii) Consider the area under the curve $y = \ln x$ from $x = 1$ to $x = n$.



Consider the sum of the areas of the upper rectangles

$$= f(2)\times 1 + f(3)\times 1 + \ldots + f(n-1)\times 1 + f(n)$$

$$= \ln 2 + \ln 3 + \ldots + \ln n$$

$$= \ln\{2.3\ldots n\}$$

$$= \ln\{1.2.3\ldots n\}$$

$$= \ln(n!)\ \text{u}^2$$

Sum of area of lower rectangles < area under the curve < sum of area of upper rectangles

$$\therefore \ln((n-1)!) < n\ln n - n + 1 < \ln(n!)$$

Terminology used needed to be correct ~ clearly stating sum of areas, for clarity of solution.

(iv) As $y = e^x$ **is an increasing function**, and since

$$\ln((n-1)!) < n\ln n - n + 1 < \ln(n!),$$

<table>
<tr><td>MEX12-5</td><td>

then $e^{\ln((n-1)!)} < e^{n\ln n-n+1} < e^{\ln(n!)}$

$\therefore (n-1)! < e^{\ln n^n} e^{-n+1} < n!$

$\therefore (n-1)! < n^n e^{1-n} < n!$

Needed to include justification for validity of inequality raised to power of *e* as question said 'prove'.

</td><td>1 mark: correct solution</td></tr>
<tr><td>MEX12-5</td><td>

(b) (i)

$I_n = \int_0^1 x^n \sqrt{1-x}\,dx = \left[\frac{-2x^n(1-x)^{\frac{3}{2}}}{3}\right]_0^1 - \int_0^1 \frac{-2(1-x)^{\frac{3}{2}}\,nx^{n-1}dx}{3}$

where $u = x^n$, $du = nx^{n-1}dx$, $dv = (1-x)^{\frac{1}{2}}$, $v = \frac{-2(1-x)^{\frac{3}{2}}}{3}$

$\therefore I_n = 0 - \int_0^1 \frac{-2(1-x)^{\frac{3}{2}}\,nx^{n-1}dx}{3}$

$\therefore I_n = \frac{2n}{3}\int_0^1 (1-x)^{\frac{3}{2}} x^{n-1}dx$

$\therefore I_n = \frac{2n}{3}\int_0^1 (1-x)(1-x)^{\frac{1}{2}} x^{n-1}dx$

$\therefore I_n = \frac{2n}{3}\int_0^1 \left\{1(1-x)^{\frac{1}{2}} x^{n-1} - x(1-x)^{\frac{1}{2}} x^{n-1}dx\right\}$

$\therefore I_n = \frac{2n}{3}\left\{\int_0^1 x^{n-1}\sqrt{1-x}\,dx - \int_0^1 x^n\sqrt{1-x}\,dx\right\}$

$\therefore I_n = \frac{2n}{3}\{I_{n-1} - I_n\}$

$\therefore 3I_n = 2nI_{n-1} - 2nI_n$

$\therefore (2n+3)I_n = 2nI_{n-1}$

$\therefore I_n = \frac{2n}{2n+3}I_{n-1}$

(ii) $\int_0^1 x^3\sqrt{1-x}\,dx = I_3$

$= \frac{6}{9}I_2$

$= \frac{6}{9}\times\frac{4}{7}I_1$

$= \frac{6}{9}\times\frac{4}{7}\times\frac{2}{5}I_0$

$= \frac{16}{105}\int_0^1 x^0\sqrt{1-x}\,dx$

$= \frac{16}{105}\int_0^1 (1-x)^{\frac{1}{2}}\,dx$

</td><td>

3 marks : correct solution

2 marks: substantial progress towards correct solution

1 mark : significant progress towards correct solution

2 marks : correct solution

1 mark : significant progress towards correct solution

</td></tr>
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MEX12-2	$=\frac{16}{105}\left[\frac{-2}{3}(1-x)^{\frac{3}{2}}\right]_0^1$ $=\frac{16}{105}\times\frac{2}{3}$ $=\frac{32}{315}$ (iii) $I_n=\frac{2n}{2n+3}I_{n-1}$ $=\frac{2n}{2n+3}\times\frac{2(n-1)}{2n+1}I_{n-2}$ $=\frac{2n}{2n+3}\times\frac{2(n-1)}{2n+1}\times\frac{2(n-2)}{2n-1}I_{n-3}$ $=\frac{2n}{2n+3}\times\frac{2(n-1)}{2n+1}\times\frac{2(n-2)}{2n-1}\times\frac{2(n-3)}{2n-3}\times\ .\ .\ .\ \times\frac{6}{9}\times\frac{4}{7}\times\frac{2}{5}\times\frac{2}{3}$ $=\frac{2^n\times n(n-1)(n-2)\times...\times3\times2\times1\times2}{(2n+3)(2n+1)(2n-1)\times...\times7\times5\times3}$ $=\frac{2^{n+1}n!}{(2n+3)(2n+1)(2n-1)\times...\times7\times5\times3}\times\frac{(2n+2)(2n)(2n-2)\times...\times6\times4\times2}{(2n+2)(2n)(2n-2)\times...\times6\times4\times2}$ $=\frac{2^{n+1}n!\times2(n+1)2(n)2(n-1)\times...\times2(3)\times2(2)\times2(1)}{(2n+3)(2n+2)(2n+1)(2n-1)\times...\times3\times2\times1}$ $=\frac{2^{n+1}n!\times2^{n+1}(n+1)!}{(2n+3)!}$ $=\frac{n!(n+1)!\left(2^{n+1}\right)^2}{(2n+3)!}$ $=\frac{n!(n+1)!\left(2^2\right)^{n+1}}{(2n+3)!}$ $\therefore I_n=\frac{n!(n+1)!\,4^{n+1}}{(2n+3)!}$ **This could also be done by Induction.** **Clarity of the solution was required for full marks.**	3 marks : correct solution 2 marks: substantial progress towards correct solution 1 mark : significant progress towards correct solution